Primordial black hole production due to preheating

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During the preheating process at the end of inflation the amplification of field fluctuations can lead to the amplification of curvature perturbations. If the curvature perturbations on small scales are sufficiently large, primordial black holes (PBHs) will be overproduced. In this paper we study PBH production in the two-field preheating model with quadratic inflaton potential. We show that for many values of the inflaton mass m, and coupling g, small scale perturbations will be amplified sufficiently, before backreaction can shut off preheating, so that PBHs will be overproduced during the subsequent radiation dominated era.

I. INTRODUCTION

There has recently been great interest in the reheating process which occurs at the end of inflation. It has been found that in many inflation models reheating proceeds initially via a period of broad parametric resonance, known as *preheating* [1,2]. During preheating the inflaton field decays extremely rapidly producing an exponentially growing number of particles, until the backreaction of the particle production shuts off the parametric resonance with the particles produced subsequently thermalising and the universe becoming radiation dominated.

Attention has been focused on two-field models with either a quadratic [2] or a quartic inflaton potential [3,4]. It has been claimed in Ref. [5] that the amplification of curvature perturbations could be large with fluctuations being driven non-linear, even on large 'super-Hubble' scales. This would have serious consequences for cosmology, destroying the standard picture of structure formation [6], in particular by violating the microwave background constraint on the amplitude of the power spectrum on large scales, and leading to the overproduction of primordial black holes (PBHs) on small scales. The growth on large scales, relevant for structure formation, has been confirmed for the quartic case [7–9], whilst it has recently been shown for the quadratic inflaton potential [10–12], that in fact the amplification of the fluctuations on large scales is negligible. The formation of PBHs has not been studied for either model. For the quadratic potential the power spectrum of curvature perturbations has a k^3-spectrum however, growing strongly towards small scales, so that PBH formation may still be problematic despite the amplification of large scale fluctuations being negligible.

In this letter we present the first study of the formation of PBHs due to the resonant amplification of field fluctuations during preheating. The scales which we are interested in pass outside the Hubble radius towards the end of inflation. The fluctuations are amplified during preheating and PBHs will be formed, when these scales re-enter the Hubble radius after reheating during the radiation dominated era, if the fluctuations are sufficiently large. PBHs may in principle also be formed on the slightly shorter scales which re-enter the Hubble radius during preheating, when the universe is dominated by the oscillating scalar fields, however 3-D simulations along the lines of the 1D simulations carried out in Ref. [13] would be necessary to study this.

II. CALCULATION OF THE POWER SPECTRUM

In common with Refs. [5,12] we will study the two-field model with quadratic scalar field potential

$$V(\phi, \chi) = \frac{1}{2} m^2 \phi^2 + \frac{1}{2} g^2 \phi^2 \chi^2 \,, \qquad (1)$$

where ϕ is the inflaton and χ the preheating field, m the mass of the inflaton and g the coupling of the inflaton to the preheat field. The ϕ field, and hence the effective mass of the χ field, $m_\chi = g\phi$, oscillates around zero with large amplitude resulting in efficient preheating. The power spectrum of the curvature perturbation in the constant density gauge, ζ, produced in this model has recently been calculated by Liddle, Lyth, Malik and Wands [12] (LLMW hereafter). The scales which we are interested in have $k \ll g\Phi$, where Φ is the initial amplitude of the ϕ-field oscillations, and are outside the Hubble radius at the end of inflation, so that we can use the results of their calculation. In this section we outline the relevant results of LLMW, for further details see Ref. [12].

Strong parametric resonance, resulting in large amplification of the initial quantum fluctuation in the χ field, occurs if

$$q \equiv \frac{g^2 \Phi^2}{4m^2} \gg 1 \,. \tag{2}$$

The power spectrum of a quantity x is defined as [6]

$$\mathcal{P}_x \equiv \frac{k^3}{2\pi^2} \left\langle |x_{\mathbf{k}}|^2 \right\rangle \,, \tag{3}$$

where $k = |\mathbf{k}|$ is the comoving wavenumber, $x_{\mathbf{k}}$ are the coefficients of the Fourier expansion of x, and the average is over ensembles. The effect of preheating on the amplitude of the χ field can be modelled as [2,12]

$$\mathcal{P}_{\delta\chi} = \mathcal{P}_{\delta\chi}|_{\mathrm{end}} \exp\left(2\mu_k m\Delta t\right) , \tag{4}$$

where Δt is the time elapsed since the end of inflation. The power spectrum of $\delta\chi$ at the end of inflation is denoted by $\mathcal{P}_{\delta\chi}|_{\mathrm{end}}$. The Floquet index μ_k is taken as

$$\mu_k \simeq \frac{1}{2\pi} \ln\left(1 + 2e^{-\pi\kappa^2}\right) , \tag{5}$$

and

$$\kappa^2 \equiv \left(\frac{k}{k_{\mathrm{max}}}\right)^2 \equiv \frac{1}{18\sqrt{q}}\left(\frac{k}{k_{\mathrm{end}}}\right)^2 , \tag{6}$$

where k_{max} is the scale which undergoes maximum amplification and k_{end} is the comoving wavenumber of the scale which exits the Hubble radius at the end of inflation. For strong coupling ($q \gg 1$) $\mu_k \approx \mu = \ln 3/2\pi \approx 0.17$ for all modes outside the Hubble radius at the end of inflation ($\kappa \ll 1$).

LLMW find that the evolution of the curvature perturbations is due to the non-adiabatic pressure perturbation δp_{nad}, which is dominated by second-order fluctuations in χ, $g^2\phi^2\delta\chi^2$ (see also [14]). The first-order contribution, which is of order $g^2\phi^2\chi\delta\chi$, is negligible since the background field χ is vanishingly small [10,11]. The power spectrum of ζ_{nad} resulting from the amplification of the non-adiabatic fluctuations in the χ field, is then given by

$$\mathcal{P}_{\zeta_{\mathrm{nad}}} \simeq A \left(\frac{k}{k_{\mathrm{end}}}\right)^3 I(\kappa, m\Delta t) , \tag{7}$$

where

$$A = \frac{2^{9/2}3}{\pi^5\mu^2}\left(\frac{\Phi}{m_{\mathrm{Pl}}}\right)^2\left(\frac{H_{\mathrm{end}}}{m}\right)^4 g^4 q^{-1/4} , \tag{8}$$

and

$$I(\kappa, m\Delta t) \equiv \frac{3}{2}\int_0^{\kappa_{\mathrm{cut}}} d\kappa' \int_0^\pi d\theta \, e^{2(\mu_{\kappa'} + \mu_{\kappa - \kappa'})m\Delta t}\kappa'^2 \sin\theta , \tag{9}$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{k}'$ and κ_{cut} is an ultraviolet cut-off, $\kappa_{\mathrm{cut}} \sim \kappa_{\mathrm{max}}$. LLMW found that on the scales relevant for large scale structure formation the change in ζ due to preheating is negligible, however since

$\mathcal{P}_{\zeta_{\mathrm{nad}}} \propto k^3$ smaller scale fluctuations with wavenumbers $k \sim k_{\mathrm{end}}$ may become large enough for PBHs to be overproduced, before backreaction shuts off preheating.

The duration of the first stage of preheating, $m\Delta t_{\mathrm{BR}}$, during which backreaction is unimportant and fluctuations are amplified exponentially, can be estimated numerically by finding the smallest solution of [2]

$$m\Delta t \approx \frac{1}{4\mu} \ln \frac{10^6(m\Delta t)^3}{g^4\sqrt{48\pi q}} \,. \tag{10}$$

After this stage backreaction becomes important; energy is drained from the inflaton field, and transferred to the χ field, diminishing the amplitude of the inflaton oscillations. The growth in the preheating field changes the effective mass of the χ field itself, thereby rendering preheating less efficient. The growth in the χ field also changes the oscillation frequency of the inflaton, narrowing the resonance band and eventually shutting the resonance down. The duration of this second stage of preheating is typically far shorter than that of the first stage of preheating. The solution of Eq. (10), $m\Delta t_{\mathrm{BR}}$, thus provides us with a conservative estimate of the duration of the entire preheating process.

III. BLACK HOLE ABUNDANCE CONSTRAINTS

Primordial black holes may be formed in the the early universe via the collapse of sufficiently large density perturbations [15]. Due to their observational consequences there are tight constraints on their abundance; typically less than 10^{-20} of the energy in the universe can go into PBHs at the time that they form [16,17]. For a PBH to form during radiation domination the density contrast, δ, when the fluctuation enters the horizon must exceed a critical value δ_{c}, where numerical simulations find $\delta_{\mathrm{c}} \sim 0.7$ [18]. The fraction of the energy of the universe which goes into PBHs is found by integrating the probability distribution of the fluctuations, $p(\delta)$, over the large, PBH forming, fluctuations,

$$\beta = \frac{\rho_{\mathrm{pbh}}}{\rho_{\mathrm{tot}}} \approx \int_{\delta_{\mathrm{c}}}^\infty p(\delta)\mathrm{d}\delta \,, \tag{11}$$

and is controlled by the mass variance, $\sigma(k)$, at horizon crossing, of the fluctuations:

$$\sigma^2(k) = \int_0^\infty \delta^2 p(\delta)\mathrm{d}\delta \,. \tag{12}$$

The density perturbation distribution is close to gaussian; strictly speaking it is a chi-squared distribution with a large number of degrees of freedom, resulting from the convolution of the square of the gaussian distributed
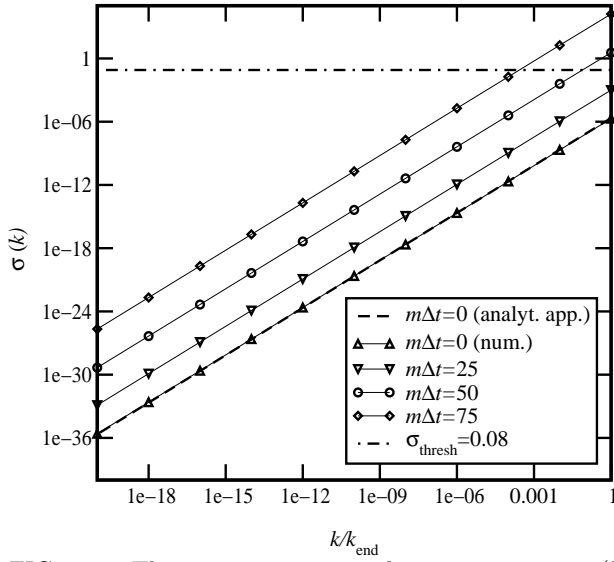
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FIG. 1. The mass variance at horizon crossing, $\sigma(k)$, versus dimensionless wavenumber k/k_{end}, where k_{end} is the comoving Hubble radius at the end of inflation, for $m\Delta t = 0, 25, 50$ and 75, for inflaton mass $m = 10^{-6}$ and coupling $g = 10^{-3}$. The dot-dashed line shows the threshold $\sigma_{\text{thresh}} = 0.08$ above which PBHs will be overproduced.

FIG. 2. The minimum preheating duration for which PBHs are overproduced, $m\Delta t_{\text{PBH}}$, versus the coupling g for $q = 10^6$ and 10^9. The dot-dashed lines show the time at which the first stage of preheating ends for the respective values of q. This shows that the fluctuations are amplified sufficiently, before backreaction sets in, for PBHs to be overproduced subsequently.

field fluctuations over a wide range of scales.[*] We find, using Eqs. (11) and (12), that for a gaussian probability distribution PBHs are overproduced, $\beta > 10^{-20}$, if $\sigma(k) > \sigma_{\text{thresh}} = 0.08$. For comparison for a first order chi-squared distribution the threshold would be $\sigma_{\text{thresh}} = 0.03$. We use $\sigma_{\text{thresh}} = 0.08$ as a conservative estimate of the threshold for PBH overproduction. To calculate $\sigma(k)$ the power spectrum must be smoothed using a window function, $W(kR)$, [6]:

$$\sigma^2(k) = \frac{16}{81} \int_0^\infty \left(\frac{\tilde{k}}{k}\right)^4 \mathcal{P}_{\zeta_{\text{nad}}}(\tilde{k}) W^2(\tilde{k}R) \frac{d\tilde{k}}{\tilde{k}} . \quad (13)$$

The precise form of the window function is not important and we take it to be a gaussian,

$$W(\tilde{k}R) = \exp\left(-\frac{\tilde{k}^2 R^2}{2}\right), \quad (14)$$

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[*]This is significantly different from the case of single field models where difficulties arise in calculating β [19]. In those models PBH production requires large field fluctuations, which are non-linear and therefore may not be gaussian distributed. Furthermore there is no definite prescription for relating the field and metric fluctuations if the field fluctuations are large. This is not the case in our model as the leading order metric perturbations arise from terms which are second-order in the *linear* field fluctuations.
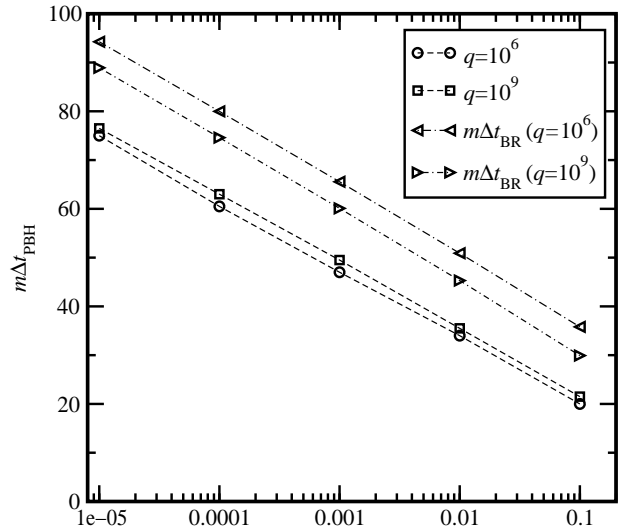
where $R = 1/k = 1/aH$, is the scale of interest, i.e. the Hubble radius at the time the PBHs form.

For $m\Delta t = 0$, i.e. immediately after inflation ends before preheating commences, $I(\kappa, 0) = \kappa_{\text{cut}}^3 \sim 1$, so that we can calculate $\sigma(k)$ approximately analytically. Inserting Eq. (7) into Eq. (13) and defining the dimensionless wavenumber $\tilde{\kappa} = \tilde{k}/k_{\text{max}}$ we find

$$\sigma^2(k)\big|_{m\Delta t=0} \approx \frac{16}{81} A \left(\frac{k_{\text{max}}}{k}\right)^4 \left(\frac{k_{\text{max}}}{k_{\text{end}}}\right)^3 \times \quad (15)$$
$$\int_0^\infty \tilde{\kappa}^6 \exp\left[-\left(\frac{k_{\text{max}}}{k}\right)^2 \tilde{\kappa}^2\right] d\tilde{\kappa} .$$

The integral in Eq. (15) above can be evaluated as

$$\int_0^\infty \tilde{\kappa}^6 \exp\left[\left(\frac{k_{\text{max}}}{k}\right)^2 \tilde{\kappa}^2\right] d\tilde{\kappa} = \frac{15}{16}\sqrt{\pi} \left(\frac{k}{k_{\text{max}}}\right)^7 , \quad (16)$$

so that the mass variance squared at the beginning of preheating is given by

$$\sigma^2(k)\big|_{m\Delta t=0} \approx 10\sqrt{2\pi} A q^{\frac{3}{4}} \left(\frac{k}{k_{\text{end}}}\right)^3 , \quad (17)$$

which provides a check for our numerical results below.

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IV. RESULTS

We evaluate Eq. (7) numerically using the values of the Hubble parameter H and the inflaton ϕ at the end of slow roll inflation: $H_{\mathrm{end}} = m/3$ and $\Phi = \phi_{\mathrm{end}} = m_{\mathrm{Pl}}/\sqrt{12\pi}$.

In Fig. 1 we plot the mass variance at horizon crossing, $\sigma(k)$, for several values of $m\Delta t$, versus the dimensionless wavenumber k/k_{end}, for the commonly used parameter values $m = 10^{-6} m_{\mathrm{Pl}}$, $g = 10^{-3}$; for these values $q \approx 6600$. For $m\Delta t = 0$ we reproduce the analytic expression found above, Eq. (17). We find that for $m\Delta t \sim 35$ the mass variance $\sigma(k)$ exceeds $\sigma_{\mathrm{thresh}} = 0.08$ on small scales, leading to the overproduction of PBHs. Since $\sigma(k)$ is largest on small scales, PBHs are formed most easily immediately after preheating. In this preheating model $k = aH \propto a^{-1/2}$ and the scale factor grows by at most ~ 16 [2] so that k changes by, at most, a factor of 0.25 during preheating. For definiteness we therefore evaluate $\sigma(k)$ on the scale $k_\star = 0.25 k_{\mathrm{end}}$. This approach is conservative; we over-estimate the Hubble radius immediately after preheating and consequently also over-estimate the preheating duration neccesary to over-produce PBHs.

In Fig. 2 we plot $m\Delta t_{\mathrm{PBH}}$, the minimum preheating duration for which the mass variance on small scales exceeds the threshold for PBH overproduction, $\sigma(k_\star) > \sigma_{\mathrm{thresh}} = 0.08$, versus the coupling g, for two values of the parameter q, $q = 10^6$ and 10^9. For comparison we also plot the estimate of the duration of preheating, $m\Delta t_{\mathrm{BR}}$, found from Eq. (10). Similarly in Fig. 3 we plot $m\Delta t_{\mathrm{PBH}}$ and $m\Delta t_{\mathrm{BR}}$ as a function of g for $m = 10^{-6} m_{\mathrm{Pl}}$ and $10^{-9} m_{\mathrm{Pl}}$. The minimum preheating duration for which PBHs are overproduced, $m\Delta t_{\mathrm{PBH}}$, depends most strongly on the strength of the coupling between the inflaton and preheating fields, g, decreasing as the coupling increases. We find that, for all sets of parameters investigated, the field fluctuations are amplified sufficiently during preheating that PBHs are overproduced during the subsequent radiation dominated era. The minimum preheating duration for which PBHs are overproduced is at least $m\Delta t \sim 25$ smaller than than the duration of the first stage of preheating, during which backreaction is unimportant and the fluctuations undergo exponential amplification.

V. DISCUSSION

In this letter we have investigated primordial black hole formation in the two-field preheating model with a quadratic inflaton potential. Whilst the amplification of the field fluctuations on the scales relevant for structure formation is negligible [10–12], the power spectrum of curvature perturbations has a k^3-spectrum, i.e. the fluctuations are largest on small scales. We find that the
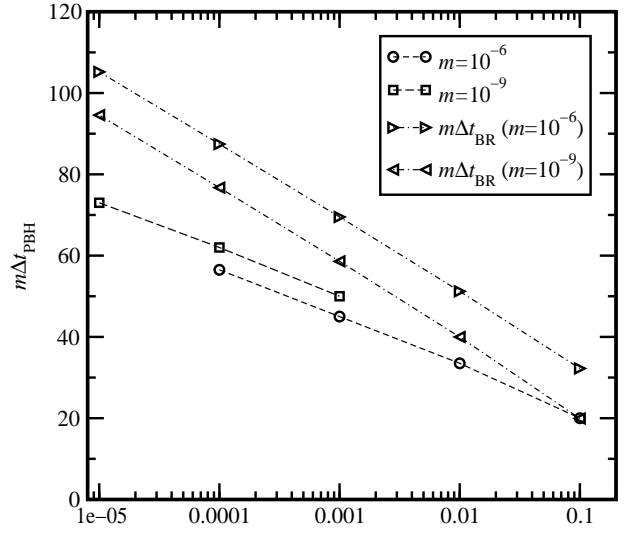

FIG. 3. The minimum preheating duration for which PBHs are overproduced, $m\Delta t_{\mathrm{PBH}}$, versus the coupling g for two values of the inflaton mass, $m = 10^{-6} m_{\mathrm{Pl}}$ and $10^{-9} m_{\mathrm{Pl}}$. The dot-dashed lines show the time at which the first stage of preheating ends for the respective masses. This shows that the fluctuations are amplified sufficiently, before backreaction sets in, for PBHs to be overproduced subsequently.

fluctuations are amplified sufficiently during preheating, before backreaction is expected to shut-off the resonance, for PBHs to be overproduced during the subsequent radiation dominated era. We emphasize that, in the strong preheating regime, the overproduction occurs irrespective of the strength of the coupling of the inflaton to the preheat field. This constitutes a serious problem for this preheating model.

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